CHAPTER 6 THE ECONOMIC CONCEPT OF BENEFIT

INTRODUCTION

Consider the question: "what is the value of the wetlands of San Francisco Bay?". Why might such a question be addressed to an economist rather than a philosopher or a poet? To explain this it is vital to distinguish between two different meanings that might be attached to the original question: (i) How much value do people place on the wet ands (assuming an adequate base of information)? (ii) How much value ought they to place on them? The latter question is certainly the province of the phi osopher or the poet; the economist too may have some thoughts about the question, but these arise from his private sentiments, not from his professional discipl inc. The former question - the positive question - is the one that the discipl ine of economics addresses. When we talk of benefits and benefit measurement in this report, we have this interpretation in mind - the values that people actual ly place on ecosystems.

This itself raises a host of questions: Which people? In what units should values be measured? Why do people have these values? Just how do we ascertain them? We will comment briefly on each of the first three questions. The answers to the fourth question will take up the remainder of this chapter, as well as Chapter 7. <u>Which people</u>? This is specif ed, in principle, by the agency commissioning the benefit assessment. A rela ed, and more comp ex question, is: How do we add up different peep e's va ues? Again, this is specified, in principle, by the agency commissioning the study; however, here there is a body of economic theory which can guide the answer - see, for example, Sen (1973), Blackorby and Donaldson (197(3), 2nd Boadway and Bruce (1984, Chapter 9). To save space, we will duck this issue here. <u>What units?</u> Values can be measured in monetary units or in units of any commodity that people happen to value. For example, we could measure the value to an individual of aquatic ecosystems in units of chocolate truffles - Lake Tahoe is worth 100 truffles, say, while Mono Lake is worth only 82 truffles. Different systems of units will generate the same ordinal ranking of ecosystems, but not necessarily the same cardinal index of value. We choose to adopt money - purchasing power - as our unit of measurement because this is the predominant convention. It is possible to develop an analogous theory of benefit measurement based on chocolate truffle units, but we shall not explore this here (aggregation across individuals would presumably be more difficult').

How do we ascertain values? In principle there are two ways to proceed: (i) Ask people directly, and (ii) Rely on revealed preference - observe their behavior when they make choices on which the aquatic ecosystem somehow impinges, and infer their values from this behavior. In this chapter we focus on the latter approach exclusively. An immediate implication is an answer to the question: Why do people have these values? The answer is that it doesn't matter. We rely on preferences as revealed by actual behavior, without needing to know how these prefences might be decomposed into alternative motives. Or rather, there are two circumstances in which motives might matter. The first is when a knowledge of motives gives us reason to believe that preferences (and behavior) will be different in the future. Stability of preferences is essential to extrapolation from observed behavicr. If preferences are not stab l e, this poses both a philosophical and a practical problem. The philosophical problem is: Which set of preferences do we rely on? The practical problem is: How can we predict what the new preferences will be if it is decided to rely on them? The other circumstance in which we might care about motives has to do with aggregation across individuals: specifically, a knowledge of motives may help us to identify groups of individuals who have different preferences. For empirical purposes, it might be more appropriate to analyze the behavior of each group separately, rather than to aggregate them into a single group.

Given the focus on revealed preference, why is the presence of markets required for the success of our endeavor? One answer common among non-economists, but erroneous, is that values are embodied in market prices and expenditures. Markets are needed because market prices establish values: if a commodity sells for $10, that is the value of the commodity. However, this is not exactly true. If I buy the commodity at a price of $10, then it certainly must be worth $10 to me - but it may be worth even more; i.e., the price is a lower bound on value. If I do not buy the commodity at this price, it is not worth $10 to me; i.e., the price is an upper bound. Let us switch from prices to expenditures and focus on the first case. Suppose I buy 5 units of the commodity at the going price of $10, so that my total expenditure is $50. This expenditure is clearly a lower bound on the value of the commodity to me. The problem, however, is that this lower bound may be inadequate for our purposes. Ultimately we are interested in net benefits - i.e. benefits minus costs. If the cost of supplying the commodity is also $10 a unit, the cost amounts to $50 and the difference between that and our lower bound estimate of benefits is zero - because we underestimate benefits when we use expenditures, we underestimate net benefits, possibly to the point of absurdity. Moreover, consider some change in the supply of the commodity (for example an improvement in its quality) which leads me to spend $70 on it. For the same reason as before, this $70 is a lower bound on the value of the improved commodity to me. But <u>the change in expenditure conveys absolutely no information about the change in value:</u> the difference between two lower bounds is not necessarily a lower bound on the difference in the quantities being bounded.

In short, we do not care about markets because market expenditures directly indicate values. At best they provide bounds on values, but these bounds are frequently so imprecise as to be useless, and the changes in market expenditures provide no information about changes in values. Instead, we care about markets because they provide a forum for choice behavior - performing tradeoffs between goods and money - from which we can indirectly infer preferences. That is the essence of the revealed preference approach. Moreover, as will be shown in the next section, these market transactions, or tradeoffs, can convey information about preferences for other items of value which are not themselves traded in a market, as long as the preferences for the latter items interact (in a sense to be made specific below) with preferences for the traded items. We turn, now, to an elaboration of this argument.

THE BASIC FRAMEWORK

The revealed preference approach to benefit assessment can be explained in terms of two basic consumer choice models. Both models pertain to an individual consumer - we want to avoid the complications associated with estimation and interpretation of aggregate demand functions. In the first model, the individual has preferences for various marketed commodities, whose consumption is denoted by the vector x, and for various environmental resources, which are denoted by q: this could be a vector but, for simplicity of notation, we treat it as a scalar. These preferences are represented by a utility function $u(x,q)$ which is continuous and non-decreasing in all arguments (we assume that the x's and q are all "goods"), and strictly quasi concave in x (we assume strict quasiconcavity rather than quasiconcavity in order to rule out demand correspondences). At this point, we do not assume that $u(\ )$ is (strictly) quasiconcave in q. The individual chooses his consumption of the marketed goods - the x's - by maximizing his utility subject to a budget constraint

$$\max_{x} \quad u(x,q) \quad \text{s.t.} \quad \sum p_i x_i = y\,, \quad x_i \geq 0 \quad i = 1..N \tag{1}$$

where the p_i 's are the prices of the marketed goods, and y is the individual 's income. Note that he does not determine the level of the q variables. These are in the nature of public goods for him, and he takes them as given.

The utility maximization generates a pattern of consumption behavior represented by the ordinary demand functions $x_i = h^i(p, q, y)$ $i = 1, \ldots, N$. For convenience we assume that these represent an interior solution, so that problems associated with corner solutions (discussed in Bockstael, Hanemann, and Strand [1984, Chapter 9] can be ignored. Substitution of these demand functions into the direct utility function yields the indirect utility function $v(p,q,y) \equiv u[h(p,q,y), q]$. Alternatively, as a dual to (1) there is an expenditure minimization problem

$$\min_x \sum p_i x_i \quad \text{s.t.} \quad u(x,q) = u, \quad x_i \geq 0 \tag{2}$$

which yields a set of compensated demand functions, $x_i = g^i(p,q,u)$, and the expenditure function $m(p,q,u) \equiv \sum p_i g^i(p,q,u)$.

These constructs can be employed to define what we mean by the benefits to the individual from a change in q. Suppose that q changes from q^0 to q^1, while prices and income remain constant at (p,y). Accordingly, the individual's welfare changes from $u^0 \equiv v(p,q^0,y)$ to $u^1 \equiv v(p,q^1,y)$. Two alternative measures of this welfare change are the quantities C and E defined, respectively, by

$$v(p, q^1, y - C) = v(p, q^0, y) \tag{3}$$

$$v(p, q^1, y) = v(p, q^0, y + E). \tag{4}$$

Each of these represents an adjustment to the individual's income calculated to offset the effects of the change in q. C, the compensating variation, is the amount of money by which the individual's income must be adjusted after the change in order to render him as well off as he was before the change. If $u^1 < u^0$, so that $C < 0$, this is the minimum compensation that he would require in order to acquiesce in the change. Similarly, E, the equivalent variation, is the amount of money by which the individual's income must be adjusted before the change in order to render him as well off as he would be after it. If $u^1 > u^0$, so that $E > 0$, this is the minimum compensation that he would require in order to forego the change while, if $u^1 < u^0$ so that $E < 0$, this is the most he would be willing to pay to avoid the change.

The second model is based on the household production approach, in which the individual gains utility from "composite commodities" which he produces himself from private goods. One version of this model is

$$\max_{x,z} w(x,z) \quad \text{s.t.} \quad f(x,z,q)=0\,, \quad \Sigma p_i x_i = y \qquad (5)$$

where z denotes the vector of composite goods, f(.) is the production function for these goods written in implicit form, and w(.) is a utility function defined over the z's and, perhaps, some of the x's. In this formulation we are assuming that the individual derives utility from q not directly, but indirectly, in so far as they contribute to the production of z's. The utility maximization in (5) can be solved in two stages. In the first stage one obtains

$$u(x,q) \equiv \max_{z} w(x,z) \quad \text{s.t.} \quad f(x,z,q)=0, \qquad (6)$$

while in the second stage one solves (1) using the function u(x,q) derived from (6). That is to say, a household production model can always be "collapsed" into a model in the form given in (1). Moreover, welfare measures for changes in q can be defined as in (3) and (4) using the indirect utility function v(p,q,y) associated with u(x,q) in (6). One consequence of the household production approach, however, is that it generates demand (and supply) curves for the z's - as well as demand curves for the x's - which are of some empirical as well as theoretical interest.

Given this framework, our analysis will be concerned with three sets of issues that have arisen in the literature on environmental benefit evaluation; (i) What is the relation between C and E - we know they must have the same sign, but how much can they differ in magnitude? (ii) How can we measure C and E from observed demand behavior - after all, since we do not observe utility directly we cannot estimate the indirect utility function $v(p,q,y)$ directly? (iii) Is there any relation between C or E and expenditures on some of the private goods - the x's - which might be specially related to the q's in terms of either consumer preferences or household production technology? Can we use expenditure on some goods as proxies for C or E?

To answer these questions, it is convenient to consider three possible markets. One is the market for x's, in which there are observable demand curves. The second is the market for z's, which may arise in connection with the household production model (5). The third market is entirely hypothetical. Suppose that the individual could actually buy q in a market at some given price, π. Instead of (1) he would now solve

$$\max_{x,q} \; u(x,q) \quad \text{s.t.} \quad \sum p_i x_i + \pi q = y \tag{7}$$

(at this point we assume strict quasiconcavity of u(.) with respect to q in order to ensure an interior solution). Denote the resulting ordinary demand functions for the x's by $\hat{h}^i(p,\pi,y)$, and the ordinary demand function for q by $\hat{h}^q(p,\pi,y)$. The associated indirect utility function is denoted by $\hat{v}(p,\pi,y) \equiv u[\hat{h}(p,\pi,y), \hat{h}^q(p,\pi,y)]$. Similarly, we could define a dual expenditure minimization problem analogous to (2), in which both the x's and q are the choice variables. The resulting compensated demand functions are denoted by $\hat{g}(p, \pi, u)$ and $\hat{g}^q(p, \pi, u)$, and the expenditure function is $\hat{m}(p, \pi, u) \equiv \sum p_i \hat{g}^i(p, \pi, u) + \pi \hat{g}^v(p, \pi, u)$.

These utility maximization and expenditure minimization problems are hypothetical because, in fact, environmental quality, q, is not a marketed commodity. Nevertheless, they are of theoretical interest because they shed light on the solutions to (1), (2), (5), and (6). For example, it is convenient to introduce the following:

DEFINITION: q is normal (inferior) if $\hat{h}^v_y > 0$ (< 0).

(We shall now adopt the convention of using subscripts to denote derivatives.)

Maler (1974) proved:

PROPOSITION 1: Assume that, if elements of q change, they all change in the same direction. Then, if all of the q's which change are normal (inferior), $C \leq E$ $(C \geq E)$

Moreover,

PROPOSITION 2: If $\hat{h}^v_y = 0$ for all the q's which change, $C = E$.

Suppose, however, that there <u>are</u> income effects in the demand functions for q; the question remains: just how much can C and E differ? To answer this, we must investigate the q-market in more detail.

HOW MUCH CAN C AND E DIFFER?

Willig (1976) established that, unless the income elasticity of demand for a commodity is very high, the compensating and equivalent variations for a price change will not differ considerably. Some environmental economists do not believe that the same holds true of compensating and equivalent variations for change in q - see, for example, Maler (1985, p.39) or Knetsch and Sinden (1984), who present empirical evidence of a considerable disparity between C and E. However, Randall and Stoll (1980) have shown that Willig's analysis carries over to changes in fixed parameters such as the q's, and Brookshire, Randall and Stoll (1980) have interpreted this result as implying that C and E should not be very different in value. How can these divergent views be explained or reconciled?

In the paper reproduced in the Appendix to this chapter I reexamine randall and Stoll's analysis and show that, while it is indeed accurate, its implications have been misunderstood. There is no presumption that C and E must be close in value and, unlike price changes, the difference between them depends not only on an income effect but also on a substitution effect. Specifically, the magnitude of the difference depends on (i) the magnitude of the change in q, (ii) the size of the income effects, and (iii) the degree of substitutability between private consumption activities (the x's) and the level of environmental quality q in the individual's preferences, all of which are empirical issues. Moreover, I suggest that the substitution effects are likely to exert far greater leverage, in practice, on the relation between C and E than the income effects. Thus, large empirical divergences between C and E may be indicative not of some failure in the survey methodology but of a general perception on the part of the individuals surveyed that the private market goods available in their choice set are, collectively, a rather imperfect substitute for the public good under consideration.

MEASURING C AND E FROM DEMAND CURVES

Analysis of the market for q is useful in that it gives us an idea of the factors that affect the relation between C and E, but it is of no value when it comes to measuring C or E in practice because, by definition, no such market exists - the demand curve for q can never be observed. What can be observed is behavior in the x market - the market for private goods. This raises the question, therefore, of whether the values of C and E can be inferred from knowledge of the demand curves from the x's. There are two ways in which this can be accomplished. The first is to uncover the indirect utility function from the fitted demand curves for the x's, and then employ the formulas in (3) and (4). The second is based upon results developed by Maler (1971, 1974) which establish a relation between areas under demand curves for the x's and the quantities C and E.

In the first approach one postulates a specific functional form for either the direct utility function $u(x,q)$ or the indirect utility function $v(p,q,y)$, and derives the appropriate formula for the corresponding ordinary demand functions - by analytically solving the direct utility maximization problem or by differentiating the indirect utility function and applying Roy's Identity. Alternatively, one can start out with a given system of ordinary demand functions $h^i(p,q,y)$ $i=1,\ldots,N$, and then attempt to recover the corresponding indirect utility function by applying the integrability techniques developed by Hurwicz and Uzawa. As a simple example, suppose that N=2 and the demand function for the first good takes the semi-log form

$$\ln x_1 = \alpha - \beta(p_1/p_2) + \gamma(y/p_2) + \delta q; \tag{8}$$

in Hanemann (1980a, 1981) it is shown that the indirect utility function is

$$v(p_1,p_2,q,y) = -\frac{e^{-\gamma(y/p_2)}}{\gamma} + \frac{Ae^{\delta q - \beta(p_1/p_2)}}{\beta} \tag{9}$$

where $A \equiv e^{\alpha}$. Application of (3) and (4) yields the following formulas for C and E

$$C = -\frac{p_2}{\gamma}\ln\left\{1 + \frac{\gamma}{\beta}(x_1^1 - x_1^0)\right\} \tag{10a}$$

$$E = \frac{p_2}{\gamma}\ln\left\{1 - \frac{\gamma}{\beta}(x_1^1 - x_1^0)\right\} \tag{10b}$$

where $x^0 \equiv h(p,q^0,y)$ and $x^1 \equiv h(p,q^1,y)$. Thus, to estimate C and

E one first fits the demand function (8) and then substitutes the estimated values of the coefficients α, β, γ and δ into the formulas in (10a,b).

The alternative approach to computing C and E′ developed by Maler, is based on the following decomposition of the formula for C (a similar analysis applies to E)

$$C = y - m(p, q^1, u^0)$$

$$= \{m(p, q^0, u^0) - m(\tilde{p}, q^0, u^0)\} - \{m(p, q^1, u^0) - m(\tilde{p}, q^1, u^0)\} + \{m(\tilde{p}, q^0, u^0) - m(\tilde{p}, q^1, u^0)\}$$

$$= \int_p^{\tilde{p}} \sum_i [g^i(p, q^1, u^0) - g^i(p, q^0, u^0)] dp_i + \{m(\tilde{p}, q^0, u^0) - m(\tilde{p}, q^1, u^0)\} \qquad (11)$$

where $\tilde{p}$ is an arbitrary price vector. Assuming that $q^1 > q^0$, we know that C > 0. Since $m_q \leq 0$, we also know that the second term in (11) is non-negative. The first term is the sum of areas between compensated demand curves corresponding to q^1 and q^0, between the actual price pi and the i^{th} element of $\tilde{p}$ (this line integral is path-independent). It should be emphasized that the first item is not necessarily positive; it can be shown that the increase in q raises the compensated demand for the i^{th} private good ($\partial g^i/\partial q > 0$) if this good is a complement to q in the Hicks-Allen sense, and lowers the compensated demand ($\partial g^i/\partial q < 0$) if the good is a substitute. Moreover, if q is a scalar, at least one of the private goods must be a Hicks-Allen substitute for q. Nevertheless, we know that the sum of the two terms in (8) must be positive.

Maler's trick is to select $\tilde{p}$ in such a way that the second term in (11) vanishes. For this purpose, he introduces two assumptions. The novel assumption is that there exists a set of commodities with the property that, if these commodities are not being consumed, the marginal utility of q is zero. Let I be the index set of these commodities, and $\bar{I}$ its complement. Partition the vector x accordingly: $x = (x_I, x_{\bar{I}})$. Maler's assumption, which he calls weak complementarily, is:

(WC) There exists a non-empty set I such that $\dfrac{\partial u(0, x_{\bar{I}}, q)}{\partial q} = 0$ (12)

His second assumption is:

(NE) The commodities in I are non-essential: there exists some price vector such that $g^i(.) = 0$ and $h^i(.) = 0$ all $i \in I$.

We can now apply these assumptions to (11) by choosing the price vector $\tilde{p}$ so that $\tilde{p}_i = p_i$ for $i \in \bar{I}$ while, for $i \in I$, $\tilde{p}_i$ is simply the cut-off price of the i^{th} compensated demand function - i.e. $\max[g^i(\tilde{p}_I, p_{\bar{I}}, q^0, u^0), g^i(\tilde{p}_I, p_{\bar{I}}, q^1, u^0)] = 0$. Since sign (m.)= - sign (u.), this yields Maler's result:

PROPOSITION 3; If $u(x,q)$ satisfies (WC) and (NE),

$$C = \int_{p_I}^{\tilde{p}_I} \sum_{i \in I} \left[g^i(p, q^1, u^0) - g^i(p, q^0, u^0) \right] dp_i . \qquad (13)$$

This proposition establishes a relationship between C and the areas between two sets of compensated demand functions. It is useful here to make a distinction between two sets of circumstances: (i) there is a set of goods with the property that q has no value only when <u>none</u> of these goods is being consumed, and (ii) there is a set of goods with the property that q has no value when any one of them is not being consumed. In the first case, C is measured by the area between compensated demand curves summed over all of the goods in I; in the second case it is measured by the area between compensated demand curves for any one of the goods in I, and we obtain the same answer regardless of the particular good selected. Note that, in order to make use of the proposition, one still needs to know something more than ordinary demand functions unless there are no income effects in the demand for the goods in I, in which case the compensated and ordinary demand functions coincide. If there are income effects and one attempts to calculate the area in (13) using ordinary instead of compensated demand functions, i.e. one calculates the area

$$S = \int_{p_I}^{\tilde{p}_I} \sum_{i \in I} \left[h^i(p, q^1, y) - h^i(p, q^0, y) \right] dp_i , \qquad (14)$$

this is likely to be of limited value. The issue is examined in Hanemann (1980b), where it is shown that under some circumstances S may not even have the correct sign. The requirement that one employ the compensated demand function in (13) implies that, wherever there are income effects, Maler's method for calculating C and E has the same information requirements as the method based on direct application of (3) and (4). Finally, as an illustration, it turns out that semi-log demand function (8) satisfies the WC condition since, on differentiating the indirect utility function (9), one finds that

$$\lim_{p_1 \to \infty} \frac{\partial v(p, q, y)}{\partial q} = 0, \tag{15}$$

which is equivalent to (12). The compensated demand function corresponding to (13) is

$$x_1 = g^1(p, q, u) = \frac{\beta}{\gamma}\left[1 - \frac{u\beta}{\gamma A} e^{\beta(p_1/p_2) - \delta q}\right]^{-1} \tag{16}$$

and it is straightforward to verify that (10a) and (10b) combine to satisfy (13).

THE LIMITS TO REVEALED PREFERENCE

Both of the methods for measuring C and E from observed demand functions rely on the assumption that all the relevant components of the indirect utility function can be recovered from demand functions. However, that assumption is not always true: it holds when the underlying direct utility function has the form

$$u = \bar{u}(x, q) \tag{17}$$

as has implicitly assumed up to now, but not when the utility function can be cast into the form

$$u = u(x,q) = T[\bar{u}(x,q), q]$$

where T(.) is increasing in its first argument and $\bar{u}(x,q)$ is a conventional direct utility function. It can be shown that both utility models imply exactly the same ordinary demand functions for x's

$$\arg\max_x \bar{u}(x,q) = \arg.\max_x T[\bar{u}(x,q), q]$$

even though they imply different things about the individual's preferences. The crucial feature of (18) is that the marginal rates of substitution between the x's - The indifference map for the x's - is independent of the transformation function T(.), even though that function influences the way in which q affects the individual's utility. This does not arise in the case of the utility function in (17). Thus , with (17), all aspects of the individual's preferences for q are captured directly or indirectly in his ordinary demand functions for x's. This is not so for (18): some aspects of the individual's preferences for q are not reflected in his ordinary demand functions, not even indirectly.

Another way of making the same point is to observe that the compensating variation for a change from (p,q^0,y) to (p,q^1,y) associated with $u(x,q)$, C, can be decomposed into two elements

$$C = \bar{C} + C^*, \tag{19}$$

where $\bar{C}$ satisfies $\bar{v}(p, q^1, y-\bar{C}) = \bar{v}(p, q^0, y)$, $\bar{v}(\ .\ )$ being the indirect utility function corresponding to $\bar{u}(x, q)$, and C^* satisfies

$$T[\bar{v}(p, q^0, y - C^*), q^1] = T[\bar{v}(p, q^0, y), q^0]. \qquad (20)$$

Assuming that $q^1 > q^0$ and $T(\ .,q)$ is increasing in q, it can be shown that $C^* > 0$, so that

$$c > \bar{C} > o. \qquad (21)$$

A similar result can be shown to hold for equivalent variation measures:

$$E = \bar{E} + E^* > \bar{E} > 0, \qquad (22)$$

where E is the true equivalent variation associated with the full utility function $u(x,q)$ in (18), $\bar{E}$ is the equivalent variation associated with the sub-function $\bar{u}(x,q)$, and E^* is calculated from the transformation function $T(\cdot,q)$, along the lines of (20). Since $\bar{C}$ and $\bar{E}$ are derived from the sub-function containing the interactions between the x's and q, we can regard them as the "consumption - or use - related" components of benefits. Similarly, we can regard C^* and E^* as the "non-consumption related" or "non-use related" components of benefits - they arise from that part of the individual's preferences which do not affect his choice of x.

The practical implications of (18) for the revealed preference approach - the measurement of C and E on the basis of observed demands for the x's - are highly important. If we only have data on ordinary demand functions for the x's, we can only recover $\bar{u}(x,q)$, but never $T(\cdot,q)$ nor the full utility function $u(x,q)$ in (18). That is, we can only measure $\bar{C}$ and $\bar{E}$ - not C* or E* and, therefore, not the full value of C or E. This is a significant limitation to the revealed preference approach.

It is sometimes thought that Maler's Weak Complementarity (WC) assumption eliminates this problem, but I would dispute this. Differentiate (18) to obtain the marginal utility of q.

$$\frac{\partial u(x,q)}{\partial q} = \frac{\partial T}{\partial \bar{u}} \frac{\partial \bar{u}}{\partial q}(x,q) + \frac{\partial T}{\partial q}. \tag{23}$$

If we apply WC to $u(x,q)$, this requires that

$$x_I = 0 \Rightarrow \frac{\partial \bar{u}}{\partial q}(0, x_{\bar{I}}, q) = 0 \text{ and } \frac{\partial T}{\partial q}[u(0, x_{\bar{I}}, q), q] = 0. \tag{24}$$

But , by itself, this is not enough to ensure that

$$\frac{\partial T}{\partial q}[\cdot, q] \equiv 0, \tag{25}$$

which is what one requires in order to rule out the representation in (18). Suppose, for example, that

$$u(x,q) = \begin{cases} \bar{u}(x,q) & \text{if } x_I = 0 \\ T[\bar{u}(x,q),q] & \text{if } x_I > 0. \end{cases} \tag{26}$$

This satisfies (24) but not (25), and therefore C* > 0 and E* > 0. In this case WC does not eliminate the problem.

To summarize, the only circumstance in which the revealed preference approach to the measurement of C and E is fully satisfactory is when (25) holds - i.e. the utility function is represented by (17) rather than (18). But there is no way to verify this from data on ordinary demand functions for x's. It <u>could</u> be verified if there were a market for q and one could observe demand functions for q as well as the x's. Indeed, in that case, T(.,q) could be recovered along with u(x,q) so that, if (25) were violated, C and E could still be calculated because one would obtain the full indirect utility function associated with (18). But , in the absence of a market for q, the problem remains.

In practice, there are two possible solutions. The first is simply to <u>assume</u> that the utility function takes the form of (17) and not (18) - which is what is generally done. The second is to collect additional behavioral data besides ordinary demand functions for the x's. For example, after measuring $\bar{C}$ by the revealed preference approach one could conduct interviews to elicit the willingness to pay for an improvement in q directly; if the interviews yielded an estimate close to $\bar{C}$ in value one would conclude that $C^* = 0$ and hence, the utility model corresponds to (17) rather than (18). If they yielded an estimate much greater than $\dot{C}$ one would take the difference to be a measure of c^*. Alternatively, instead of contingent valuation exercises, one could conduct what has been called [Hanemann (1985)] "contingent behavior" exercises in which one attempts to elicit a hypothetical demand function for q. Both of these approaches remain subjects for future research.

THE SIGNIFICANCE OF EXPENDITURE DATA

In the theory of the welfare measurement of price changes it is well known that calculation of expenditure changes provide <u>bounds</u> on the compensating and equivalent variations, even if they are not exactly equal to these welfare measures. If prices change from p^0 to p^1 and the quantities demand change correspondingly from x^0 to x^1, then the compensating variation for the price change, C^p, and the equivalent variation, E^p, satisfy

$$C^p \geq \sum_i (p_i^0 - p_i^1) x_i^0$$

and

$$E^p \leq \sum_i (p_i^0 - p_i^1) x_i^1$$

although, in general, there is no determinate relation between C or E and the overall change in expenditure $\Sigma p_i^0 x_i^0 - \Sigma p_i^1 x_i^1$.

When dealing with changes in q, as opposed to price changes, some authors have wondered whether one can obtain a relation between the welfare measures C and E and the change in expenditures on some or all of the private market goods, $\Sigma p_i [h^i(p, q^0, y) - h^i(p, q^1, y)]$. In general, I do not believe that this is a useful approach; with one exception described below, there does not appear to be any determinate relation between changes in expenditure on x's and either C and E. Indeed, the effect of an increase in q on the demand function for any of the x's is by no means obvious. Given that $(\partial u/\partial q) > 0$, it is sometimes assumed that $\partial h^i/\partial q \geq 0$ all i - an increase in quality can never lower the demand for any of the x's. In fact, this is not true; in general, an increase in q will affect the demand for the x's, but note that the effect could be in either direction, depending on the specifics of the utility function. Even if q is a Hicks-Allen complement with some private good -say, xl - it is not necessarily true that an increase in q will raise the demand for that good.

This pessimistic conclusion is based on the following proposition which links the demand functions $x_i = h^i(p,q,y)$ to the hypothetical demand functions $x_i = \hat{h}^i(p,\pi,y)$ associated with the utility maximization problem (7):

PROPOSITION 4: Let $\pi = \hat{\pi}(p,q,y)$ be defined implicitly by Appendix equation (11). Then,

$$h^i(p,q,y) = \hat{h}^i[p, \hat{\pi}(p,q,y), y + \hat{\pi}(p,q,y)\cdot q] \quad i=1,\ldots,N. \tag{27}$$

It follows as a corollary that

$$\begin{aligned} \frac{\partial h^i}{\partial q}(p,q,y) &= \hat{\pi}(p,q,y)\cdot\frac{\partial \hat{h}^i}{\partial y} + \left[\frac{\partial \hat{h}^i}{\partial \pi} + q\frac{\partial \hat{h}^i}{\partial y}\right]\cdot\frac{\partial \hat{\pi}}{\partial q}(p,q,y) \\ &= \hat{\pi}\cdot\frac{\partial \hat{h}^i}{\partial y} - \left\{\frac{\partial \hat{h}^i/\partial \pi + q\,\partial \hat{h}^i/\partial y}{\partial \hat{h}^q/\partial \pi + q\,\partial \hat{h}^q/\partial \pi}\right\}\left(\hat{\pi}\ \frac{\partial \hat{h}^q}{\partial y} - 1\right). \end{aligned} \tag{28}$$

Given that $u_q > 0$, $\hat{\pi} > 0$. If $u(x,q)$ is quasiconcave in q, the denominator of the second term on the RHS is negative. Thus, the sign of $\partial h^i/\partial q$ depends upon a complex set of factors. The numerator of the term in braces on the RHS will be recognized as the cross-price derivative of the compensated demand curve from q

$$\frac{\partial \hat{g}^q}{\partial p_i}(p,\hat{\pi},u) = \frac{\partial \hat{g}^i}{\partial \pi}(p,\hat{\pi},u) = \frac{\partial \hat{h}^i}{\partial \pi} + q\frac{\partial \hat{h}^i}{\partial y}$$

and this is positive or negative according as x_i and q are substitutes or complements. Moreover,

$$\hat{\pi}\frac{\partial \hat{h}^q}{\partial y} - 1 = \omega\eta - 1 \gtrless 0 \quad \text{as} \quad \eta \gtrless \frac{1}{\omega}$$

where $\omega \equiv \frac{\hat{\pi}q}{y}$, Thus, if $\partial \hat{h}^q/\partial y > 0$ and

$$[\omega\eta - 1]\ \frac{\partial \hat{g}^q}{\partial p_i} > 0 \tag{29}$$

this is a sufficient condition for $\partial h^i/\partial q > 0$. Even if $\partial \hat{h}^i/\partial y < 0$, it can still happen that $\partial h^i/\partial q > 0$ if (29) holds and that term is sufficiently large.

Without belaboring it further, the point is that an increase in q could either lower or raise the expenditure on x. This should make us cautious about expected any simple relation between the change in expenditure on some of the x's and C or E since it is quite possible that C and E are positive while the change in expenditure is negative. One case in which more definitive results can be obtained is where q is a perfect substitute for some of the x's - say x_1 . In that case the direct utility function takes the form

$$u(x,q) = \bar{u}[x_1 + \psi(q), x_2, \ldots, x_N] \tag{30}$$

where $\psi(\,.\,)$ is some increasing function of q. Let $\bar{h}^1(p,y)$ and $\bar{v}(p,y)$ be the ordinary demand function for good 1 and the indirect utility function associated with $\bar{u}(\,.\,)$. The following may be shown:

PROPOSITION 5: If u(x,q) has the form given in (30),

$$h^1(p,q,y) = -\psi(q) + \bar{h}^1[p_1, \ldots, p_N, y + p_1\psi(q)] \tag{31a}$$

$$v(p,q,y) = \bar{v}[p_1, \ldots, p_N, y + p_1\psi(q)]\,. \tag{31b}$$